Written by Alan Friedman, DLI Engineering
Monday, 01 September 2003 09:03

Introductory overview illustrates how awareness of the behavior of linear and nonlinear systems provides fuller understanding of machine health when analyzing vibration data.

The analysis of a vibration spectrum of a machine in the context of linearity and nonlinearity provides an additional basis for understanding why spectra look as they do and how the appearance of a spectrum relates to machine health. Here is an overview of the concept, augmented with straightforward illustrations and examples.

Linear systems

If a linear system is thought of as a black box, it can be said that what comes out of the box is directly proportional to what goes in. This concept is called proportionality. In Fig 1. we can see that the output motion is directly related to the input force. If the input force increases, the resulting motion also increases proportionally (click for Figs. 1-8).

Another quality of linear systems is superposition as demonstrated in Fig. 2. Superposition means that if we have two or more input forces, the output motion will be proportional to the sum of the input forces. In other words, nothing new is created. If we add a whole bunch of forces at the input, the output motion will still be directly proportional to the sum of those forces.

Nonlinear systems

Consider a dense metal cube sitting on ice. If you push the cube, it will slide proportionally to how hard you push it. This is a linear response. Now consider that the cube is made out of gelatin. When you give the gelatin a push it may slide a bit, but it also will wiggle and wobble. This is an example of a nonlinear response. The gelatin does not move only in the direction of the push, it also wiggles around in different directions. Therefore, we can say that the output motion is not directly proportional to the input force and therefore the gelatin block is nonlinear (Fig. 3).

Nonlinear systems also do not follow the law of superposition. This means that the output response is not proportional to the sum of the input forces. In a nonlinear system, the inputs combine with each other and produce new things in the output that were not present in the input (Fig. 4).

When one plays a stereo at a relatively low volume, the music comes out clearly. If one raises

the volume slightly, the music comes out of the speaker more loudly, but still sounds good. This is a linear response.

We reach a point, however, where if we make the stereo loud enough, the music becomes distorted, and we begin to hear new sounds that were not recorded on the CD. This is a nonlinear response. The key again to understanding when something is nonlinear is that the output contains things that were not present in the input.

Linearity and nonlinearity in vibration

Now that we have described the basic concepts of linearity and nonlinearity, it is time to discuss them in terms of vibration signals. Simple mass-spring systems as shown in figures 5 and 6 will be used for this discussion.

An ideal mass-spring system (Fig. 5) can be described by the equation

$$F = kX$$

where F is the input force, k is the spring stiffness, and X is the resulting displacement of the spring. This is a linear system. If we input a sinusoidal force, the resulting displacement is also sinusoidal and proportional to the input.

If the stiffness of the spring changes as it is stretched and compressed (Fig. 6), the system is nonlinear. When we input a sinusoidal force, the resulting displacement is not sinusoidal, and thus this is a nonlinear system in which we get out something that looks different from what we put in.

If we remember the basic rules of vibration and the Fast Fourier Transform, the displacement sine wave in Fig. 5 will produce a single peak in a vibration spectrum. The displacement wave in Fig. 6 will produce a peak in the spectrum with harmonics or multiples. This brings up another important point—the harmonics in this case are the result of nonlinearity.

Machinery vibration

When we look at the vibration spectra for a machine in the context of linear and nonlinear systems, we can make a very general statement that as machines deteriorate and develop faults they become less linear in their responses. We also can say that many machine faults create nonlinearity. Therefore, also in very general terms, we can expect the spectra from a healthy machine to be relatively simple compared with the spectra from a machine with faults. If we consider mechanical looseness as a common machine problem, we can demonstrate this.

When the machine is not experiencing looseness and is in good health, its spectra may look like that in Fig. 7, which shows the shaft rate peak (the big one on the left) and a couple of harmonics of the shaft speed. The same machine with a looseness problem (Fig. 8) might show considerably more shaft rate harmonics at higher amplitudes. This is very similar to the example of the two mass-spring systems in that when the mass-spring system was linear, only one peak was produced in the spectrum, i.e., the output looked like the input. When the mass-spring system was nonlinear, the output waveform was not sinusoidal and therefore produced harmonics in the spectrum.

If we take a step back, we can consider that the mechanical input forces in a simple rotating machine are coming from the rotating shaft. If the shaft is rotating perfectly (i.e., there is no looseness) and the response of the machine structure is perfectly linear, then we would expect to see only a single peak in our spectrum corresponding to the shaft rate. In other words, the output would look like the input. No machines are perfect, however, and shafts do not typically rotate perfectly around their centers; this is why we expect to see some harmonics in machine spectra (Fig. 7). However, as the machine becomes more nonlinear, due to a condition such as looseness, foundation cracks, or broken mounting bolts, more harmonics with higher amplitudes appear (Fig. 8).

Note that if one views a spectrum with a linear amplitude scale, one may not see the harmonic content of the spectrum if the harmonics are much smaller in amplitude than the shaft rate peak. If one views the data using a logarithmic amplitude scale, more harmonic content will be visible on the graph.

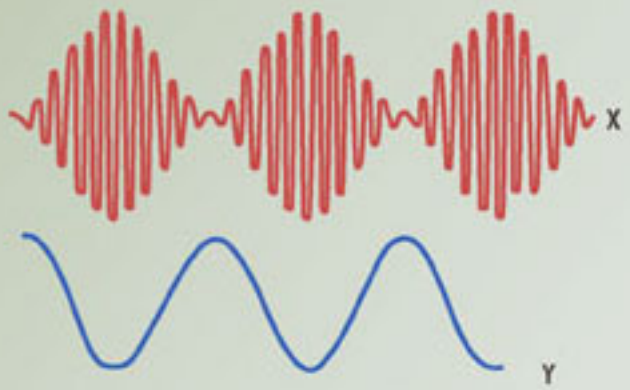


Fig. 9. Wave repeats itself at frequency X; however, amplitude of wave varies at frequency Y.

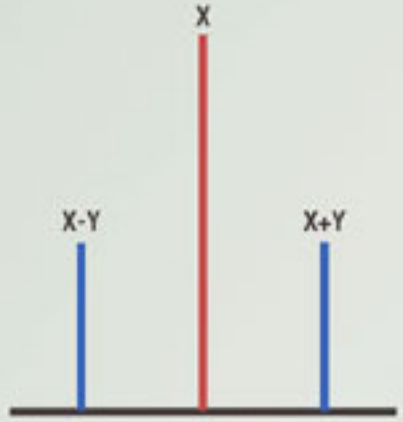


Fig. 10. Sidebands of X-Y and X+Y appear to either side of peak at frequency X.

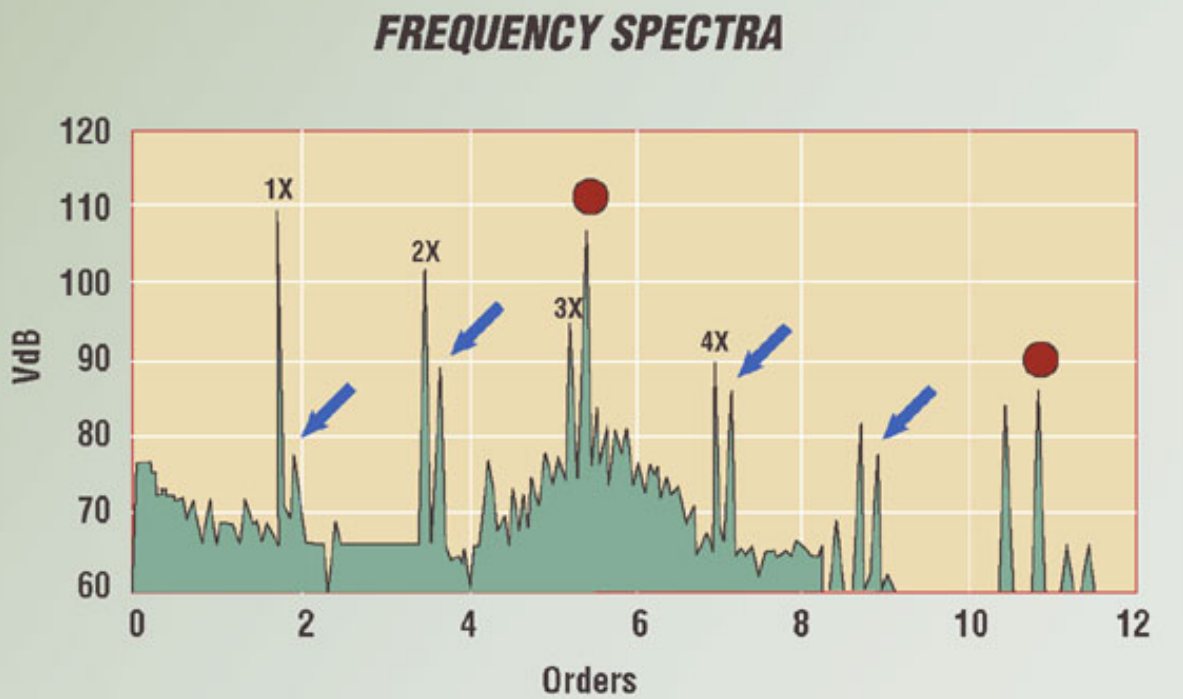


Fig. 11. Peaks with circles are nonsynchronous bearing tones. Arrows point to synchronous sidebands of shaft speed harmonics.

[redacted] DLI Engineering [redacted]

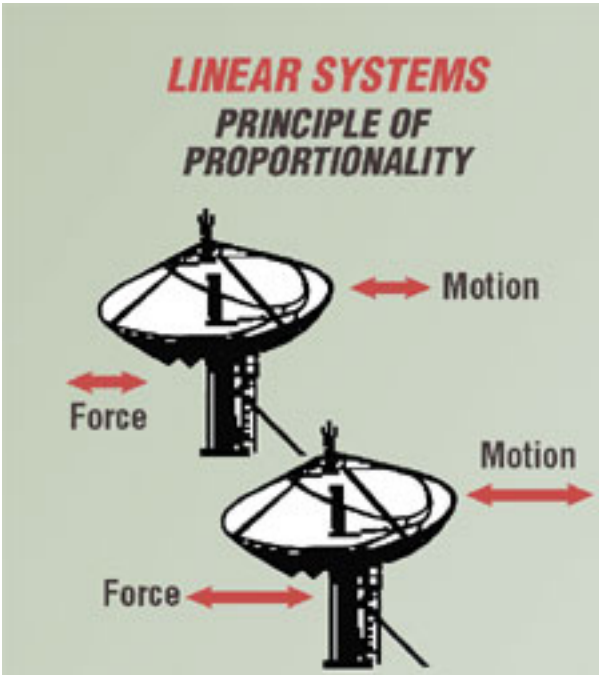


Fig. 1. Output motion is proportional to input force in a linear system.

Fig. 2. Output motion is proportional to sum of multiple input forces in a linear system.

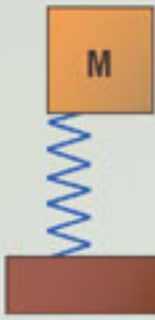


Fig. 5. Sin
produces

Fig. 3. Output motion is not proportional to input force in a nonlinear system.

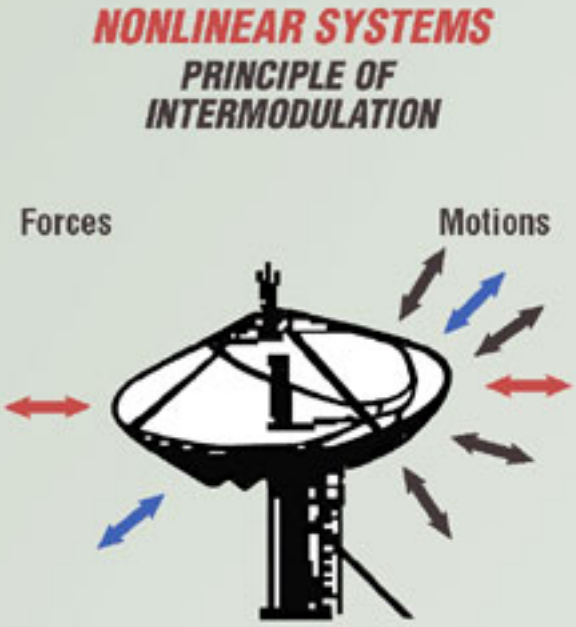


Fig. 4. Output motion is not proportional to sum of multiple input forces in a nonlinear system..

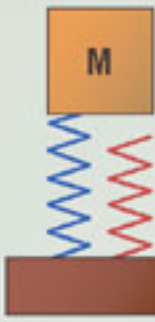
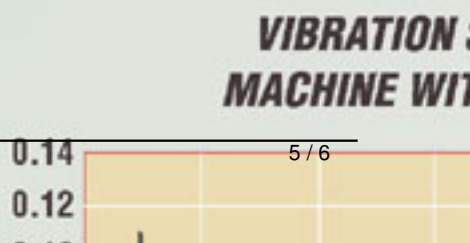


Fig. 6. Sin
system pr



return to article